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Radiative Corrections to High-Energy Inelastic Electron Scattering*

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A method for calculating radiative corrections to high-energy inelastic electron-nucleus scattering is presented. The method is applicable to the case where only the scattered electron is detected at a given angle and energy. As an example of this procedure, we have calculated the radiative corrections to inelastic electron-deuteron scattering.

I. INTRODUCTION

IN this paper, we present a general method for calculating radiative corrections to inelastic electron-nucleus scattering. The experimental situation that we consider is that of a beam of electrons with a well-defined energy being scattered through an angle θ, the energy spectrum of the final electrons being measured. In addition to the elastic peak, a continuous spectrum of inelastic electrons is observed, as shown in Fig. 1.

The diagrams which contribute to the radiative correction are shown in Fig. 2. Figure 2(a) represents the basic process without radiative corrections, Fig. 2(b) is the electron vertex modification, Figs. 2(c) and 2(d) represent the emission of a real photon, and Fig. 2(e) gives the contribution due to vacuum polarization. The "blob" in each of the figures represents the interaction of a virtual photon with the target nucleus.

Since for elastic scattering the kinematics are determined, the maximum energy of a photon which may be emitted in process 2(c) or 2(d) is limited by the experimental quantity ΔE, shown in Fig. 1. Several authors[1,2] have calculated radiative corrections to elastic electron scattering, the results being expressed in terms of the initial and final electron energies, the scattering angle, and the experimental resolution ΔE.

For the continuous inelastic spectrum shown in Fig. 1, one cannot define experimentally a quantity analogous to ΔE, hence the maximum energy of a photon which may be emitted in process 2(c) or 2(d) may be quite large. In addition to this complication, the matrix element for inelastic scattering is usually a rapidly varying function of the momentum transfer, and one must consider the change in the momentum transferred to the nucleus due to the emission of a photon in either process 2(c) or 2(d). This is in contrast to the elastic-scattering case where the matrix element is a rather slowly varying function of the momentum transfer.

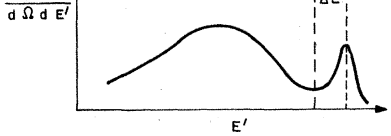

FIG. 1. Typical energy spectrum of final electrons, showing the elastic peak and the inelastic continuum.

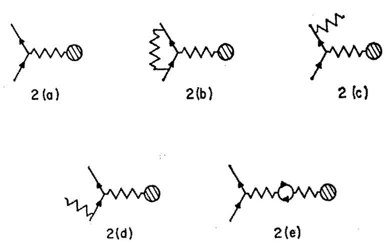

FIG. 2. Diagrams contributing to the radiative correction.

In order to simplify the calculation of the radiative corrections to the inelastic spectrum, we have made an artificial division of the photons emitted in 2(c) and 2(d) into soft and hard photons. Soft photons have an energy less than $\Delta\mathcal{E}$ while hard photons have an energy greater than $\Delta\mathcal{E}$, $\Delta\mathcal{E}$ being an arbitrary cutoff. This division simplifies greatly the calculation since one may neglect the momentum dependence of the matrix element for the basic process in calculating the soft-photon contribution, while the contribution from hard photons may be evaluated easily by assuming that photons are emitted only in two directions: namely, the directions of incoming and outgoing electrons.

The contribution of the soft photons is treated in the usual manner[3] while the contribution of the hard photons is calculated numerically, taking into account the dependence of the matrix element for the process on the momentum transfer.

In order that the calculation be valid, it must be possible to find a range of $\Delta\mathcal{E}$ where the results of the calculation are insensitive to $\Delta\mathcal{E}$. We have found that it is possible to do this for the case of inelastic electron-deuteron scattering.

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[1] Y. S. Tsai, Phys. Rev. **122**, 1898 (1961).

[2] N. Meister and D. R. Yennie, Phys. Rev. **130**, 1210 (1963).

[3] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N. Y.) **13**, 379 (1961).

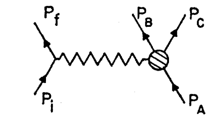

FIG. 3. Diagram representing the basic electrodisintegration process.

In the next section, we give the details of the method, while in Sec. III an example of the application of this method to the calculation of the radiative corrections to inelastic electron-deuteron scattering is given.

II. METHOD OF CALCULATION

A general formula for the uncorrected inelastic differential cross section has been given by Gourdin,[4] the result being expressed in terms of two inelastic form factors $V_0(q^2,W^2)$ and $V_1(q^2,W^2)$. For simplicity, Gourdin considered the case of electrodisintegration as shown in Fig. 3.

We use the following notation: $\hbar=c=1$, p_i and p_f are the four momenta of the initial and final electrons. $[p=(E,\mathbf{p})$ with $p^2=E^2-\mathbf{p}^2]$. p_A is the four momentum of the target nucleus A, p_B and p_C are the momenta of the recoiling particles. $q^2=(p_i-p_f)^2$, $W^2=(p_B+p_C)^2=q^2+M_A{}^2+2M_A(E_i-E_f)$.

For relativistic electrons ($E\gg m$), Gourdin's result for the diagram shown in Fig. 3 may be written in the form

$$\left(\frac{d^2\sigma}{dE_fd\Omega}\right)_0=\frac{\alpha}{2(2\pi)^3}\frac{N_AN_BN_C}{M_A}\frac{\cos^2(\theta/2)}{4E_i{}^2\sin^4(\theta/2)}\frac{P}{W}\times[V_1(q^2,W^2)+2\tan^2(\theta/2)V_0(q^2,W^2)]. \quad (2.1)$$

P is the magnitude of the three momentum of particle B in the center of momentum frame of the recoiling particles. N_A, N_B, N_C are normalization coefficients defined by $N_i=M_i$ for fermions of mass M_i, $N_i=\frac{1}{2}$ for bosons.

Although Gourdin's result is given explicitly for the case in which the target nucleus breaks down in only two particles, this form is valid for the general inelastic cross section with an arbitrary number of particles in the final state.

As discussed in the introduction, in order to obtain the radiative corrections to the inelastic cross section, we divide the photons emitted into soft photons of energy less than $\Delta\mathcal{E}$ and hard photons of energy greater than $\Delta\mathcal{E}$. The radiative corrections due to the emission of soft photons and the virtual photon contributions shown in Fig. 2(b) and Fig. 2(c) may be obtained using the results of Ref. 3. The result is

$$\left(\frac{d^2\sigma}{dE_fd\Omega}\right)_{\text{soft}}=\left(\frac{d^2\sigma}{dE_fd\Omega}\right)_0(1+\delta), \quad (2.2)$$

where

$$\delta=\frac{\alpha}{\pi}\left[\ln\left(\frac{\Delta\mathcal{E}^2}{E_iE_f}\right)\left[\ln\left(\frac{q^2}{m^2}\right)-1\right]-\tfrac{1}{2}\ln^2\left(\frac{E_i}{E_f}\right)+\frac{13}{6}\ln\left(\frac{q^2}{m^2}\right)-\frac{28}{9}\right]. \quad (2.3)$$

The radiative corrections due to the emission of hard photons are performed with the assumption that hard photons are emitted in two directions: the direction of incoming and outgoing electrons. The momentum spectrum of the hard photons is folded into the momentum dependence of the matrix element; thus the details of the interaction process are taken into account.

The T matrix for diagrams 2(c) and 2(d) is the product of two matrix elements for the electromagnetic current j_μ.

$$T_{fi}=(1/q^2)\langle p_f,k|j_\mu|p_i\rangle\langle p_Bp_C|j_\mu|p_A\rangle. \quad (2.4)$$

The first factor is

$$\langle p_f,k|j_\mu|p_i\rangle=e\bar{u}_{s'}(p_f)\gamma_\mu u_s(p_i)\left[\frac{(p_f\cdot\epsilon)}{(k\cdot p_f)}-\frac{(p_i\cdot\epsilon)}{(k\cdot p_i)}\right]+e\bar{u}_{s'}(p_f)\left[\frac{\boldsymbol{\varepsilon}\mathbf{k}\gamma_\mu}{2(k\cdot p_f)}+\frac{\gamma_\mu\mathbf{k}\boldsymbol{\varepsilon}}{2(k\cdot p_i)}\right]u_s(p_i), \quad (2.5)$$

where $u_s(p)$ is a Dirac spinor describing an electron of energy momentum p and spin s, and ϵ is the photon polarization.

We have to calculate $|T_{fi}|^2$. The summation on electron spins is easily performed and the result can be very much simplified with the assumption that photons are emitted only in the two directions previously discussed. This allows us to replace k_μ by $p_{i\mu}(k/E_i)$ for a photon emitted in the direction of incoming electrons, whenever k_μ occurs in the numerator or in a scalar product not involving $p_{i\mu}$ in the denominator. [One cannot, of course, make this approximation when k appears in the scalar product $(k\cdot p_i)$ in the denominator.] Similarly, when the photon is emitted in the direction of outgoing electrons, k_μ will be replaced by $p_{f\mu}(k/E_f)$ whenever possible. The result is

1. $\mathbf{k}/\!\!/\mathbf{p}_i$

$$\mathfrak{M}=\tfrac{1}{2}\sum_{s'}[\langle p_f,k|j_\mu|p_i\rangle\langle p_f,k|j_\nu|p_i\rangle^*]=\frac{e^2}{2m^2}\left[\left(-\frac{m^2}{(k\cdot p_i)^2}+\frac{2(p_i\cdot p_f)}{(k\cdot p_i)(k\cdot p_f)}+\frac{k}{E_i(k\cdot p_i)}+\frac{km^2}{E_i(k\cdot p_i)^2}\right)\times(p_{i\mu}p_{f\nu}+p_{i\nu}p_{f\mu}-(p_i\cdot p_f)g_{\mu\nu})-\frac{1}{(k\cdot p_i)}(p_{i\mu}p_{f\nu}+p_{i\nu}p_{f\mu}-2(p_i\cdot p_f)g_{\mu\nu})-\frac{2p_{i\nu}p_{i\mu}}{(k\cdot p_i)}+\frac{(p_i\cdot p_f)}{(k\cdot p_i)(k\cdot p_f)}\frac{k}{E_i}(2p_{i\mu}p_{i\nu}-p_{i\mu}p_{f\nu}-p_{i\nu}p_{f\mu})\right]. \quad (2.6a)$$

[4] M. Gourdin, Nuovo Cimento 21, 1094 (1961).

2. $\mathbf{k}/\!/\mathbf{p}_f$

$$\mathfrak{M}=\frac{e^2}{2m^2}\bigg[\bigg(-\frac{m^2}{(k\cdot p_f)^2}+\frac{2(p_i\cdot p_f)}{(k\cdot p_i)(k\cdot p_f)}+\frac{k}{E_f(k\cdot p_f)}-\frac{km^2}{E_f(k\cdot p_f)^2}\bigg)(p_{i\mu}p_{f\nu}+p_{i\nu}p_{f\mu}-(p_i\cdot p_f)g_{\mu\nu})$$

$$+\frac{1}{(k\cdot p_f)}(p_{i\mu}p_{f\nu}+p_{i\nu}p_{f\mu}-2(p_i\cdot p_f)g_{\mu\nu})+\frac{2p_{f\nu}p_{f\mu}}{(k\cdot p_f)}+\frac{(p_i\cdot p_f)}{(k\cdot p_i)(k\cdot p_f)}\frac{k}{E_f}(p_{f\mu}p_{i\nu}+p_{f\nu}p_{i\mu}-2p_{f\mu}p_{f\nu})\bigg]. \quad (2.6b)$$

After integration on photon angle, in order to get rid of the angular dependence in the denominators of expressions (2.6a) and (2.6b), we get the very simple result

1. $\mathbf{k}/\!/\mathbf{p}_i$

$$\int d\Omega\mathfrak{M}=\frac{2\pi e^2}{km^2}\bigg[\frac{1}{k}\bigg(1-\frac{k}{E_i}\bigg)\bigg(2\ln\bigg(\frac{2E_i}{m}\bigg)-1\bigg)+\frac{k}{E_i^2}\ln\bigg(\frac{2E_i}{m}\bigg)\bigg][p_{i\mu}p_{f\nu}+p_{i\nu}p_{f\mu}-(p_i\cdot p_f)g_{\mu\nu}], \quad (2.7a)$$

2. $\mathbf{k}/\!/\mathbf{p}_f$

$$\int d\Omega\mathfrak{M}=\frac{2\pi e^2}{km^2}\bigg[\frac{1}{k}\bigg(1+\frac{k}{E_f}\bigg)\bigg(2\ln\bigg(\frac{2E_f}{m}\bigg)-1\bigg)+\frac{k}{E_f^2}\ln\bigg(\frac{2E_f}{m}\bigg)\bigg][p_{i\mu}p_{f\nu}+p_{i\nu}p_{f\mu}-(p_i\cdot p_f)g_{\mu\nu}]. \quad (2.7b)$$

This tensor is proportional to the well-known tensor appearing in the basic process when no photon is emitted. Thus it enables us to give the contribution from hard-photon emission as a function of the same inelastic form factors that appear in the uncorrected cross section given by Gourdin. The result for the hard-photon contribution is the sum of two terms. For each case there corresponds a different momentum transfer and energy of the recoiling particles in their center-of-momentum frame. Thus, we define the following quantities:

$$(p_i-p_f-k)^2=q_i^2=q^2(1-k/E_i) \quad \text{for} \quad \mathbf{k}/\!/\mathbf{p}_i; \quad (2.8a)$$

$$(p_i-p_f-k)^2=q_f^2=q^2(1+k/E_f) \quad \text{for} \quad \mathbf{k}/\!/\mathbf{p}_f. \quad (2.8b)$$

Similarly, we define the corresponding quantities W_i, P_i, $k_{\max i}$ for $\mathbf{k}/\!/\mathbf{p}_i$ and W_f, P_f, $k_{\max f}$ for $\mathbf{k}/\!/\mathbf{p}_f$.

Our result for the hard photon contribution is

$$\bigg(\frac{d^2\sigma}{dE_fd\Omega}\bigg)_{\text{hard}}=\frac{\alpha^2}{(2\pi)^4}\frac{N_AN_BN_C}{M_A}\frac{\cos^2(\theta/2)}{4E_i^2\sin^4(\theta/2)}\bigg[\int_{\Delta\mathcal{E}}^{k_{\max i}}dk\frac{P_i}{W_i}\frac{1}{(1-k/E_i)^2}[V_1(q_i^2,W_i^2)+2\tan^2(\theta/2)V_0(q_i^2,W_i^2)]$$

$$\times\bigg[\frac{1}{k}\bigg(1-\frac{k}{E_i}\bigg)\bigg(2\ln\bigg(\frac{2E_i}{m}\bigg)-1\bigg)+\frac{k}{E_i^2}\ln\frac{2E_i}{m}\bigg]+\int_{\Delta\mathcal{E}}^{k_{\max f}}dk\frac{P_f}{W_f}\frac{1}{(1+k/E_f)^2}$$

$$\times[V_1(q_f^2,W_f^2)+2\tan^2(\theta/2)V_0(q_f^2,W_f^2)]\bigg[\frac{1}{k}\bigg(1+\frac{k}{E_f}\bigg)\bigg(2\ln\bigg(\frac{2E_f}{m}\bigg)-1\bigg)+\frac{k}{E_f^2}\ln\frac{2E_f}{m}\bigg]\bigg]. \quad (2.9)$$

The final result is given by

$$\bigg(\frac{d^2\sigma}{dE_fd\Omega}\bigg)_{\text{total}}=\bigg(\frac{d^2\sigma}{dE_fd\Omega}\bigg)_{\text{soft}}+\bigg(\frac{d^2\sigma}{dE_fd\Omega}\bigg)_{\text{hard}}. \quad (2.10)$$

As pointed out by Tsai,[5] Eq. (2.9) could, in principle, be used to determine the factors V_0 and V_1 from the experimental cross section by means of an iterative procedure; i.e., one first neglects the radiative corrections in determining V_0 and V_1. Then the radiative corrections are calculated using Eq. (2.9) and a new set of form factors is found. The iteration is continued until a consistent set of form factors is determined. In practice, we feel that it is more feasible to assume a model for V_0 and V_1 in order to calculate the radiative corrections.

It should be emphasized that the validity of this procedure is dependent on being able to find a cutoff $\Delta\mathcal{E}$ such that the total cross section is practically independent of $\Delta\mathcal{E}$ over a large range of values. This is indeed the case in our numerical results obtained for inelastic electron-deuteron scattering, given in the following section.

[5] Y. S. Tsai, Proceedings of the International Conference on Nucleon Structure, Stanford, June 1963 (to be published).

TABLE I. Radiative corrections to high-energy inelastic electron-deuteron scattering. The results were calculated using a cutoff $\Delta\mathcal{E}=5$ MeV. For simplicity of notation, we have written $(d^2\sigma/dE_fd\Omega)$ as simply σ. The quantity Δ is defined by the relation $\sigma_{\text{total}}=\sigma_0(1-\Delta)$. All differential cross sections are expressed in $\text{F}^2\ \text{MeV}^{-1}$.

E_i	E_f	θ	q^2	σ_0	σ_{soft}	σ_{hard}	σ_{total}	Δ
339	285	60	2.48	1.33×10^{-6}	1.11×10^{-6}	7.01×10^{-8}	1.18×10^{-6}	0.11
197	143	135	2.47	2.195×10^{-7}	1.90×10^{-7}	1.08×10^{-8}	2.01×10^{-7}	0.08
936	623	60	14.95	2.86×10^{-8}	2.15×10^{-8}	3.675×10^{-9}	2.52×10^{-8}	0.12
586	273	145	14.92	7.20×10^{-9}	5.70×10^{-9}	8.72×10^{-10}	6.57×10^{-9}	0.087

TABLE II. Effect of the variation of $\Delta\mathcal{E}$ on the corrected cross section. The notation is the same as that of Table I.

				σ_{total}			
E_i	E_f	θ	q^2	$\Delta\mathcal{E}=3$ MeV	$\Delta\mathcal{E}=5$ MeV	$\Delta\mathcal{E}=10$ MeV	$\Delta\mathcal{E}=15$ MeV
475	377	60	4.59	3.80×10^{-7}	3.78×10^{-7}	3.78×10^{-7}	3.80×10^{-7}
401	302.5	75	4.61	2.37×10^{-7}	2.36×10^{-7}	2.36×10^{-7}	2.37×10^{-7}
352	254	90	4.58	1.66×10^{-7}	1.65×10^{-7}	1.65×10^{-7}	1.66×10^{-7}
283	185	135	4.58	8.11×10^{-8}	8.10×10^{-8}	8.12×10^{-7}	8.17×10^{-8}

III. ILLUSTRATIVE EXAMPLE

As an example of the application of this procedure, we have calculated the radiative corrections to inelastic electron-deuteron scattering. The model we take for V_0 and V_1 in this case is that given by Durand.[6] In this model, the deuteron is assumed to be in a pure S state and to be adequately represented by a Hulthèn wave function. No final-state interaction between the final nucleons is taken into account. Although this model is perhaps not the best available, it is sufficient to give a very good first approximation to the shape of the inelastic electron spectrum, which is all that is required for the calculation of the radiative corrections. Neglecting the interference terms which give a negligible contribution, Durand's result may be written in the form

$$V_0(P,q^2)=4(4\pi)^2\alpha N^2(2M-\epsilon)(q^2/4M^2)[(F_{1p}+K_pF_{2p})^2+(F_{1n}+K_nF_{2n})^2]I(P,q^2), \quad (3.1\text{a})$$

$$V_1(P,q^2)=4(4\pi)^2\alpha N^2(2M-\epsilon)[F_{1p}^2+F_{1n}^2+(q^2/4M^2)\times(K_p^2F_{2p}^2+K_n^2F_{2n}^2)]I(P,q^2), \quad (3.1\text{b})$$

where

$$I(P,q^2)=\frac{1}{P^2q^2}\left[\frac{1}{(x^2-1)}+\frac{1}{(y^2-1)}+\frac{1}{(y-x)}\ln\left|\frac{(x-1)(y+1)}{(x+1)(y-1)}\right|\right]$$

$$x=(\alpha^2+P^2+q^2/4)/Pq;\quad y=(\beta^2+P^2+q^2/4)/Pq.$$

The Hulthèn wave function of the deuteron is

$$\Psi=\frac{N}{(4\pi)^{1/2}}\left[\frac{e^{-\alpha r}-e^{-\beta r}}{r}\right]\chi_{1,m}.$$

The proton form factors F_{1p} and F_{2p} are determined by the elastic electron-proton scattering experiments. The neutron form factors F_{1n} and F_{2n} are obtained from the inelastic electron-deuteron experiments and in principle may be determined only after the radiative corrections are known. However, the results for the radiative corrections are quite insensitive to the exact value of the nucleon form factors. In our calculation, we have used the form factors given by DeVries *et al.*[7]

Some typical results of the calculation are given in Tables I and II. Note that, from the results of Table II, a value of $\Delta\mathcal{E}$ can be found such that the total corrected cross section is independent of this parameter over a wide range.

Our method for calculating radiative corrections has been employed to calculate the corrections to the shape of the spectrum of inelastic electron-deuteron scattering. A typical result is shown in Fig. 4. Although it is difficult to estimate the errors introduced by our approximations, we believe that the over-all result is

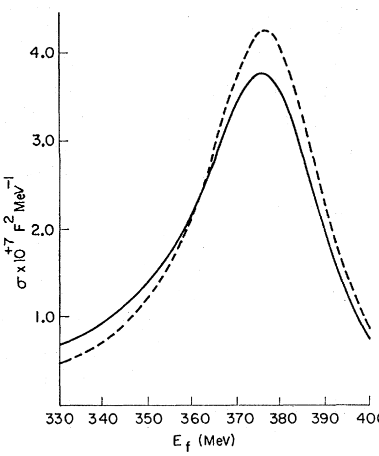

FIG. 4. Energy spectrum of final electrons following the electrodisintegration of deuterium. The incident energy is 475 MeV and the scattering angle 60°. The dashed curve shows the uncorrected spectrum based on the Durand's theory, while the solid curve shows the corrected spectrum.

[6] L. Durand, III, Phys. Rev. 123, 1393 (1961).

[7] C. DeVries, R. Hofstadter, and R. Herman, Phys. Rev. Letters 8, 381 (1962).

of the same degree of accuracy as the calculations of radiative corrections for elastic-scattering processes. We have, however, throughout this treatment neglected the emission of photons by the heavy particles. This process may become important for very energetic electrons; further calculations would be necessary in that situation.

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Construction of Amplitudes with Massless Particles and Gauge Invariance in *S*-Matrix Theory

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The fundamental statement of relativistic invariance for scattering amplitudes is that the amplitude remains invariant when the momentum and spin variables of each particle are transformed according to the corresponding irreducible, unitary representation of the inhomogeneous Lorentz group. To "construct an amplitude" is to find the most general function that has the required transformation properties. This construction, which had been previously effected for any number of massive particles of arbitrary spin, is extended here to include massless particles of arbitrary spin as well. In the case of photons, the resulting formalism is compared with the usual one that makes use of transverse polarization vectors and a gauge-invariance condition. The two formalisms are proven to be equivalent. It is concluded that the gauge condition is superfluous as an independent physical principle for the purpose of constructing amplitudes. Its use in the conventional formalism is simply a way of imposing the Lorentz-transformation properties appropriate to massless particles. In an Appendix, the known analogous construction for massive spin-one particles is shown to be equivalent to the usual formalism, and the requirement of Lorentz invariance is shown to be equivalent to the usual prescription for virtual photons as well.

I. INTRODUCTION

IN the analysis of scattering phenomena, the fundamental quantity is the scattering amplitude. It is a function of the momenta of the various incoming and outgong particles and a finite dimensional matrix in the spin space of the various particles. The total dimensionality of the amplitude is the product of the dimensionalities of the spin space of each particle, so that a particle of finite mass and spin j ($j=0, \frac{1}{2}, 1\cdots$) contributes a factor $2j+1$ to the total dimensionality, while a massless particle contributes a factor 1. Massless particles of opposite helicity are counted as different particles, since no proper Lorentz transformation, which is what relates different physical observers, mixes these states.

Each particle corresponds to an irreducible unitary representation of the inhomogeneous Lorentz group. Under a Lorentz transformation, the amplitude remains invariant when the momentum and spin variables of each particle are transformed according to the corresponding representation. This is the fundamental statement of Lorentz invariance for scattering phenomena and is expressed mathematically below. By "constructing a scattering amplitude" is meant finding the most general matrix of given dimensionality that has the correct transformation properties. In practice, this is accomplished by expressing the amplitude as a finite sum over a minimum number of spin matrices multiplied by Lorentz scalar coefficients. It is these spin matrices, with the correct transformation properties, that are actually constructed.

The reasons for basing the construction on Lorentz invariance alone are twofold. On the one hand, the method is direct and provides a unified treatment for all spins. On the other hand, it is important in the confrontation of theory with experiment to lay bare the logical foundations of the theory so that it is clear when a general postulate, such as Lorentz invariance, is being tested, rather than more-particular assumptions. In the literature, one finds most commonly an alternative method. Namely, the most general invariant operator is constructed that may be sandwiched between eigenfunctions of the free-field equations corresponding to the various scattered particles. This method is perhaps more cumbersome, since the number of field components is in general larger than the number of spin states, particularly for large spin. Also, the free field corresponding to a given spin is not unique.[1] More im-

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[1] E. P. Wigner, *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), p. 60.